## Chapter $4<$ Proportion and inverse proportion 0

## OKey points of study

1 Functions, Variables and domains $\mathrm{P} 95 \sim$
(1) Letters that can take on a variety of values are called variables.
(2) Suppose there are two variables $x$ and $y$ that change together. When you determine the value of $x$, if there is a unique corresponding value of $y, y$ is a function of $x$.
(3) When a variable is within a range of values, that range is called the domain of the variable.

## 2 Proportion and inverse proportion $\mathrm{P} 97 \sim$

(1) When $y$ is a function of $x$ and the relationship between them is expressed as $y=a x(a \neq 0), y$ is proportional to $x$ and $a$ is called the constant of proportion.
(2) When $y$ is a function of $x$ and the relationship between them is expressed as $y=\frac{a}{x}(a \neq 0), y$ is inversely proportional to $x$ and $a$ is called the constant of proportion.

## 3 Coordinates $\mathrm{P} 102 \sim$

(1) Imagine two number lines intersecting perpendicularly at point O , which is their common origin, as in the figure on the right. In this figure, the horizontal number line is called the $x$-axis, and the vertical number line is called the $y$-axis. Both $x$-axis and $y$-axis are collectively called the coordinate axes, and point O is called the origin.

(2) Point P in the figure on the right corresponds to a pair of values $x=3$ and $y=2$. They are expressed as $(3,2)$ and called the coordinates of point P. 3 is the $x$-coordinate and 2 is the $y$-coordinate of point P .
(3) The coordinates of the reflection of point $\mathrm{P}(a, b)$ about the $x$-axis are $(a,-b)$, those of the reflection of point P about the $y$-axis are $(-a, b)$, and those of the reflection of point P about the origin are $(-a$, $-b)$.
(4) The coordinates of the midpoint between $\mathrm{P}(a, b)$ and $\mathrm{Q}(c, d)$ are $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$.

4 Graphing proportion and inverse proportion P104~
(1) The graph of the proportional relationship $y=a x$ is a straight line that passes through the origin.
(2) The graph of the inversely proportional relationship

 $y=\frac{a}{x}$ is two curves called a hyperbola.



## (1) Proportion and inverse proportion

Exercises $\Rightarrow$ P101
Let's learn the basics 1 Functions
(1) Letters that can take on a variety of values are called variables.
(2) Suppose there are two variables $x$ and $y$ that change together. When you determine the value of $x$, if there is a unique corresponding value of $y, y$ is a function of $x$.

Question Suppose you cut $x$ pieces of $8-\mathrm{cm}$ ribbon from a 1-m ribbon. Letting $y$ be the length of the piece that is left uncut, answer the following questions.
(1) Express $y$ in terms of $x$. (2) Is $y$ a function of $x$ ?

Solution (1) (The length of the piece that is left uncut) $=$ (The original length) - (The sum of the lengths of the pieces that were cut off), so

$$
y=100-8 \times x \longrightarrow y=100-8 x
$$

When expressing $y$ in terms of $x$, the term containing a letter is usually placed in front of the term that is just a number. For example, $y=-8 x+100$.
(2) When you determine the value of $x$, there is a unique corresponding value of $y$. Therefore, $y$ is a function of $x$.
Answer
(1) $y=-8 x+100$
(2) Yes

1 Suppose it took $x$ hours to walk $y \mathrm{~km}$ at a speed of 5 km per hour. Answer the following questions.(1) Express $y$ in terms of $x$.
(2) Is $y$ a function of $x$ ?

2 Choose all options indicating that $y$ is a function of $x$ and answer using numbers (i) to (iv).
(i) The circumference of a square with a side of $x \mathrm{~cm}$ is $y \mathrm{~cm}$.
(ii) A person's height is $x \mathrm{~cm}$ and his weight is $y \mathrm{~kg}$.
(iii) You traveled $y \mathrm{~km}$ by taxi and paid a fare of $x$ yen.
(iv) It took $y$ minutes to empty a tank filled with 50 L of water by draining $x \mathrm{~L}$ per minute.

3 Suppose a $30-\mathrm{cm}$ candle burns to become 2 cm shorter per minute. Letting $y$ be the length of the candle $x$ minutes after it was lit, answer the following questions.
$\square(1)$ Express $y$ in terms of $x$.(2) Fill in the blanks of the following table.

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | $\cdots$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 30 |  |  |  |  |  |  | $\cdots$ |

$\square(3)$ Find the value for $y$ when $x=13$. (4) Find the value for $x$ when $y=8$.

## Co Comprehension test for Chapter 60

1 Positional relationships between lines and planes The figure on the right shows a solid created by cutting a cuboid with a plane. When $\mathrm{AE} / / \mathrm{BF}$, answer the following questions.(1) How many edges are there that are parallel to edge DC?(2) Find all edges that are in skewed positions in relation to edge DC.

$\square(3)$ Find a face that is perpendicular to edge EF.
$\square(4)$ Is there any face that is parallel to face AEHD?

2 Prisms and cylinders Find the volume and surface area of the following prisms and cylinder.

$\square(2$

$\square(3)$


3 Cones Answer the following questions about the cone on the right, noting that its bottom face is a circle with radius 8 cm , its height is 6 cm , and its generatrix is 10 cm long.(1) Find the volume of this cone.
(2) Find the central angle of the sector that makes up the side face.
$\square(3)$ Find the surface area of this cone.

4 Spheres The figure on the right shows one of the solids created by cutting a sphere into 8 equal parts. Find the volume and surface area of this solid.


5 Projections Name the solid shown in each projection below, and find its volume.
$\square(1$

$\square(2)$


## End-of-chapter problems

1 Find the volume of the solid that is created by revolving the shape below once around the axis of revolution line $\ell$.
$\square(1)$

$\square(2)$

2. Suppose you rolled a cone, whose bottom face has a radius of 5 cm , on a plane with its vertex O as the center, as shown in the figure on the right. When the edge of the cone moved on the dotted line and the cone came back to the starting point, it rotated just three times. Answer the following questions about this cone.

(1) Find the length of its generatrix.(2) Find its surface area.

3 The figure on the right shows a cube whose side is 6 cm . Point P is on side DC. When the volume of quadrangular pyramid $\mathrm{F}-\mathrm{PABC}$ is $\frac{2}{9}$ of the volume of this cube, find the length of segment PC.


43 Figure 1 on the right shows a triangular prism with AB $=3 \mathrm{~cm}, \mathrm{AC}=4 \mathrm{~cm}, \angle \mathrm{BAC}=90^{\circ}, \mathrm{BE}=6 \mathrm{~cm}$, and $\mathrm{AP}=\mathrm{BQ}$ $=\mathrm{FR}=2 \mathrm{~cm}$. Two solids are created by cutting this prism with a plane that passes through three points $\mathrm{P}, \mathrm{Q}$, and R. Figure 2 shows the one containing point D. Answer the following questions about the solid in Figure 2.


1) Find all edges that are in skewed positions in relation to edge QR .
(2) Find the volume of this solid.
